# MOCK UPCAT 6: ANSWER KEY WITH SOLUTIONS

1. **A**

$$\frac{x}{z+1} = y$$
$$x = yz + y$$
$$x - y = yz$$
$$\frac{x-y}{y} = z$$

2. **D**

Use a simple example. k = 1

a. $k^2 = 1^2 = 1$; odd

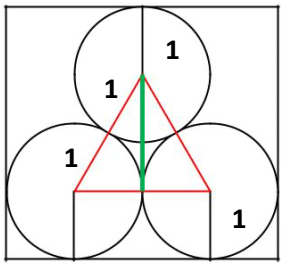
b. $k^2 + 2 = 1^2 + 2 = 3$; odd

c. $2k + 1 = 2(1) + 1 = 3$; odd

d. $2k + 2 = 2(1) + 2 = 4$; even

e. $2k + k/2 = 2(1) + (1/2) = 2.5$; not odd nor even.

3. **E**




The red triangle inside is equilateral triangle with side 2 units. The angles inside the equilateral triangle are equal to $60^o$. To get the height of the triangle (the green line), we use the 30-60-90 triangle relationship.

If the hypotenuse is 2 units, the side opposite to the $60^o$, which happens to be the height of the equilateral triangle, is equal to $\sqrt{3}$. The total height of the figure is $2 + \sqrt{3}$.

4. **E**

Let w = width, l = length, P = perimeter

$$w = -\frac{l}{2} - 2.$$

we know that P = 2w + 2l

40 = 2w + 2l

40 = 2( – 2) +2l

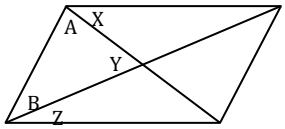
40 = l – 4 + 2l

40 = 3l – 4

44 = 3l

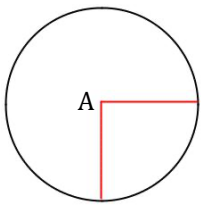
l = 44/3

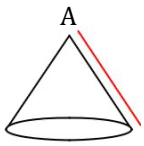
5. **E**




| Statement | Reason |
|---|---|
| 1. m∠A+ m∠B+ m∠Y = 180 | 1. Triangle Angle Sum Theorem |
| 2. m∠Z+ m∠B+ + m∠A + m∠X = 180 | 2. Consecutive Angles of a Parallelogram |
| 3. m∠A+ m∠B+ m∠Y = m∠Z + m∠B+ m∠A + m∠X | 3. Transitive Property of Equality |
| 4.m∠Y = m∠Z+ m∠X | 4. Subtraction Property of Equality |
| 5. m∠Z = - m∠X+ m∠Y | 5. Subtraction Property of Equality |

6. **D**




The area of the remaining portion of the circle is:

$$A_{new} = A_{old} = \frac{3}{4} x\, \pi\, 1^2 = \frac{3\pi}{4}$$

The surface area of the cone without a base is πrl where l is the slant height of the cone, in this case, the old r = 1.

$\pi rl = \pi \times r \times 1 = \frac{3\pi}{4}$

r = 3/4

7. **E**

Fibonacci Sequence

Start with 1 and 1.

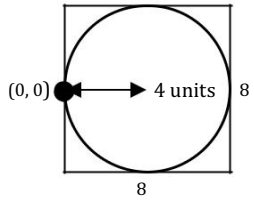
$3^{rd}$ number = $1^{st} + 2^{nd}$ =1+1 = 2

$4^{th}$ number = $2^{nd} + 3^{rd}$ =1+2 = 3

$5^{th}$ number = $3^{rd} + 4^{th}$ = 2+3 = 5

$8^{th}$ number = $6^{th} + 7^{th}$ = 8+13 = 2

8. **E**




The center of the circle lies on the x-axis, 4 units away from the origin. Thus, (4,0).

9. **B**

Let w = width, l = length, P = perimeter

$w = \frac{1}{2} + 2$

l = w + 3

we know that P = 2w + 2l

Using the equation of w and P,

$P = 2(\frac{1}{2} + 2) + 2l$

P = l + 4 + 3l

P = 4 + 3l

10. **C**

For an implication statement of the form *If P, then Q,* only the form *If not P, then not Q* is true. This is called the contrapositive of the statement. Implications and their contrapositives are equivalent.

11. **B**

$$f(x) = \frac{4x+8}{3-2x}$$

$$f(x-1) = \frac{4(x-1)+8}{3-2(x-1)}$$

$$= \frac{4x-4+8}{3-2x+2} = \frac{4x-4}{5-2x}$$

12. **B**

Let x be the number of hours they worked together.

Paolo's rate = ¼

John's rate = ½

$$\frac{x}{4} + \frac{x}{2} = 1$$

$$x + 2x = 4$$

$$3x = 4$$

$$x = \frac{4}{3} = 1\frac{1}{3} \text{ hours}$$

13. **C**

The form of the parabola $x = a(y-k)^2 + h$ where (h,k) is the vertex (x,y) of the parabola. The parabola is opening to the left, so the coefficient a of $y^2$ must be negative.

14. **C**

2x + y = -6 multiply by 3

-6x + 4y = 18

6x + 3y = -18

-6x + 4y = 18

eliminate x

7y = 0

y = 0

15. **C**

$$\frac{x^2}{x+x+x} = \frac{x^2}{3x} = \frac{x}{3}$$

16. **C**

6x + 9y = 7 multiply by 2

3x - 6y = -14 multiply by 3

12x + 18y = 14

9x - 18y = -42

eliminate y

21x = -28

x = -28/21 = -4/3

substituting x into the second equation

3(-4/3) – 6y = -14

-4 – 6y = -14

-6y = -10

y = 10/6 = 5/3

The answer is (-4/3, 5/3).

17. **B**

$5x^2y^2 + 3x^2y - 10xy - 36 + (xy(16xy - 4x + 10))$

$= 5x^2y^2 + 3x^2y - 10xy - 36 + 16x^2y^2 - 4x^2y + 10xy$

$= 21x^2y^2 - x^2y - 36$

18. **B**

Let Jericho's age be x since it has no descriptions

| | Now | + 2 years |
|---|---|---|
| Joan's age | x + 8 | x + 8 + 2 = x +10 |
| Jericho's age | x | x + 2 |

x+10 = 2 (x+2)
x+10 = 2x + 4
x = 6

19. **C**

0.0001y =1
0.0001y x 1000 = 1 x 1000; 0.1y = 1000
0.0001 y x 10000 = 1 x 10000; 1y = 10000

1y + 0.1y = 10000 + 1000 = 11000

20. **C**

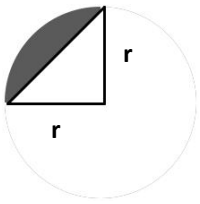
$$\frac{p+q}{p-q} = \frac{\frac{2}{3}+\frac{5}{7}}{\frac{2}{3}-\frac{5}{7}} = \frac{\frac{14+15}{21}}{\frac{14-15}{21}} = \frac{\frac{29}{21}}{-\frac{1}{21}}$$
$$= \frac{29}{21} \div -\frac{1}{21} = \frac{29}{21} x -\frac{21}{1}$$
$$= -29$$

21. **B**

The volume of the prism is equal to V = Ah where A is the area of the base. In this case, a prism with a square base has area $V = s^2h$.

$54 = s^2 \times 6$
$s^2 = 9$
s = 3

22. **B**




The area of the triangle is $\frac{r^2}{2}$.

The area of the quarter circle is $\frac{\pi r^2}{4}$.

Subtracting the area of the triangle from the area of the quarter circle, we get

$$\frac{\pi r^2}{4} - \frac{r^2}{2} = \frac{\frac{22}{7}(r^2)}{4} - \frac{r^2}{2}$$
$$= \frac{22r^2}{28} - \frac{14r^2}{28} = \frac{8r^2}{28} = \frac{2}{7}r^2$$

23. **B**

Since the first point is at (0,0), and the midpoint is at (4,2), this means that half of the line segment is 4 units to the right and 2 units upward. Thus, we need to extend it by another 4 units to the right and 2 units upward, getting (8,4).

24. **B**

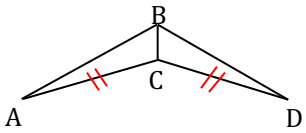
Let h be the heights and s be the lengths of the shadow
The ratio of height and length of the tree is equal to the ratio of the height and length of the stick.

$$\frac{h}{s} = \frac{1\text{ m}}{3\text{ m}} = \frac{x}{15.3\text{ m}}$$
$$x = 5.1\text{ m}$$

25. **B**

$A_{square} = s^2 = 36cm^2$
s = 6cm
$Perimeter_{square} = 4s = 4(6) = 24$ cm
$Perimeter_{square} = Perimeter_{triangle} = 24$ cm

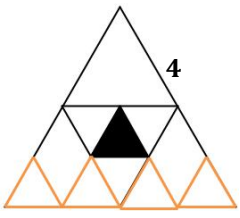
26. **C**




An angle bisector divides the angle into two equal lengths.

Since $\overline{BC}$ is a bisector of $\angle ABD$ and $\overline{AC} = \overline{CD}$,
$\Delta ABC \cong \Delta DBC$

27. **D**




It can be seen that the length of the shaded triangle is 1.

Thus, its height is equal to $\frac{\sqrt{3}}{2}$. Solving bh/2, $\frac{1 x \frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{3}}{4}$

28. **B**

Let r be the radius of the rear wheel and f be the radius of the front wheel. The relationship between the two radius is: r = 2f.

Getting the circumference of the rear wheel:

$$C_{rear} = \pi d = \pi \text{ x } 2r$$

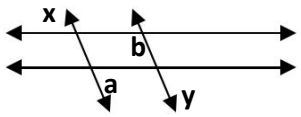
Substituting the relationship of the two wheels into the equation above,

$$\pi d = \pi \text{ x } 2r = \pi \text{ x } 2(2f) = 4\pi f$$

Thus,

$$C_{front} = \pi d = \pi \text{ x } 2f = 2\pi f$$
$$C_{rear} = 2C_{front}$$

29. **A**




| Statement | Reason |
|---|---|
| 1. m∠X = m∠A | 1. Alternate Exterior Angles |
| 2. m∠A= m∠B | 2. Alternate Interior Angle |
| 3. m∠B= m∠Y | 3. Vertical Angles |
| 4. m∠X= m∠Y | 4. Transitive Property of Equality |

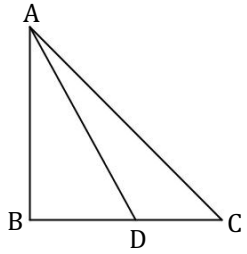
30. **D**

Using ratio and proportion, we have to add all the partitions of the ratio. 3 + 4 + 5 = 12. This corresponds to the total of the angles of the triangle, which is $180^o$. 180/12 = 15. This is the multiplier of the ratio. To get the largest angle, we should multiply 15 by the biggest partition in the ratio. 15 x 5 = 75.

31. **C**

The hypotenuse of triangle ABC is equal to $2\sqrt{2}$. If the ratio of the hypotenuse of triangle DEF to triangle ABC is $2: 2\sqrt{2}$ which can be simplified to $1: \sqrt{2} = \sqrt{2}: 2$. Since BC = 2, the length of EF is equal to $\sqrt{2}$.

32. **A**




Since the triangle is isosceles, m∠A = 45.
m∠BAD = 45 – 15 = $30^o$. To get AB and BC,

BD = AD sin 30

$$4\sqrt{3} = \text{AD}\left(\frac{1}{2}\right)$$

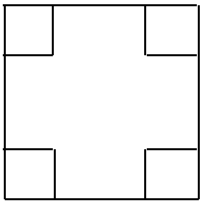
$$\text{AD} = 8\sqrt{3}$$

To get AB and BC,

$$\text{AB} = \text{BC} = \text{AD}\cos 30 = 8\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = (4\text{x}3) = 12$$

The area is equal to (12x12)/2 = 72 square units.

33. **D**



The area of the large square is 16. That means, the side of the large square is equal to $\sqrt{16} = 4$. The perimeter of each small square is equal to 4. Thus, the side of each small square is 4/4 = 1. The area of each small square is 1 x 1 = 1 square unit. 4 small squares = 4 square units. 16 – 4 = 12 square units.

34. **D**

(-3 x -6) – (-4 + -5) = 18 – (-9) = 18 + 9 = 27

35. **E**

Simplify all the values into decimal form.
a. 0.333
b. 0.600
c. 0.625
d. 0.626
e. 0.667

36. **B**

Working backwards: 5 x 20 = 100. The sum of the 5 integers is 100. 3 x 8 = 24. The sum of the middle 3 integers is 24. Thus, the sum of the first and last integer is 100 – 24 = 76. Their average is 76/2 = 38.

37. **B**

Let x be the number of girls
x + 2 be the number of boys
15 = x + (x + 2) + 5 = 2x + 7

8 = 2x
x = 4
There are 4 girls.

38. **D**
There was initially ½ V of water.
1/6 V remained after 120 mL has been removed. Thus,
1/2 V – 1/6 V = 120 mL
1/3 V = 120 mL
V = 360 mL.

39. **A**
Cars = ½ (1000000) = 500000
Bus = ¼ (1000000) = 250000
Car + Bus = 750000 = $7.5 \times 10^5$

40. **E**
The common difference is 7/12.
4/3 – ¾ = 16/12 – 9/12 = 7/12
¾ - 1/6 = 9/12 – 2/12 = 7/12
Thus, 1/6 – 7/12 = 2/12 – 7/12 = -5/12

41. **D**
15, 15, 16, 16, 17, 17,18, 18, 18, 19, 19
18 occurs 3 times.

42. **A**
In 3 hours, there are 180 minutes (3 x 60 = 180). Therefore, in 3 three hours, there are 180 x 2 = 360 people who arrived.
365-360 = 5 people initially in a party.

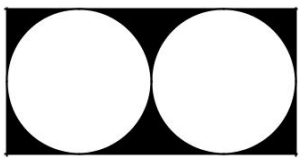
43. **B**
0.6 (4) = 0.2 + 0.8 + 1.0 + x
2.4 = 2.0 + x
x = 0.4

44. **E**

Every second, M covers 5.5m while J covers 4.5m. That's a total of 10m. Therefore, it will take 2 seconds for them to cover a total distance of 20m. At that time, M will have covered 11m.

45. **C**
**Permutation.** $\frac{n(n-1)}{2} = \frac{6(5)}{2} = 15$

46. **D**

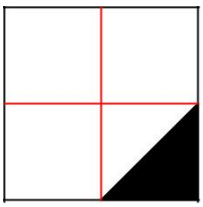


The area of the circle $9\pi = \pi r^2$
Thus, the radius of each circle is 3, and the diameter = 2r = 6. With two circles side by side, the length of the rectangle = 2d =12 and the width of the rectangle is d = 6.
The area of the whole rectangle is 12 x 6 = 72 square units.
The area of the shaded region = 72 – 2(9 π) = 72-18 π = 18(4- π).

47. **C**

The diagonal of the square = radius of the quarter circle = 6.
The area of the quarter circle = $\frac{\pi r^2}{4} = \frac{\pi \times 6^2}{4} = 9\pi$
The area of the square is $d^2/2 = 6^2/2 = 18$.
Thus, the area of the shaded region is 9π - 18.

48. **A**



The perimeter of the square is 16. Thus, its side = 16/4 = 4. Half its side is the side of the triangle. From the illustration, we can see that the triangle is ½ of ¼ of the area of the whole square. Since its side is 4, the area of the square = s x s = 4 x 4 = 16. ½ x ¼ x 16 = 2.
Or, since we know that half the side of the square is the side of the triangle, the area of the triangle is bh/2 = (2x2)/2 = 2

49. **B**
In a sequence, the $n^{th}$ term can be computed as
$a_n = a_1 + d(n-1)$
The $9^{th}$ term, $a_9 = a_1 + 8d = 9$
The $15^{th}$ term, $a_{15} = a_1+14d = 30$.
Treating the two equations as a system of equations with $a_1$ and d as the variables,
$a_1 + 8d = 9$ multiply by -1
$a_1 + 14d = 30$
$-a_1 - 8d = -9$
$a_1 + 14d = 30$

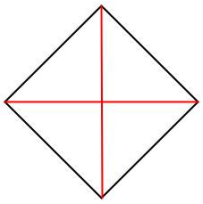
eliminating $a_1$,
$6d = 21$
$d = 3.5$

Using $a_9$ to get $a_1$,
$9 = a_1 + 8(3.5)$
$a_1 = 9 - 28$
$a_1 = -19$

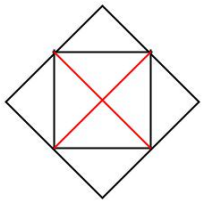
50. **C**
Let R be the radius of the bigger circle, r be the radius of the smaller circle.

$R = 3r$
If the circumference of the smaller circle is $2\pi r = 6\pi$, then the radius of the smaller circle is 3. Thus, the radius of the bigger circle is $R = 3(3) = 9$. Therefore, the circumference of the bigger circle is $2\pi r = 2\pi(9) = 18\pi$.

51. **B**

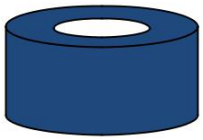


The smaller square is half the area of the biggest square.



The smallest square is half the area of the smaller square.
If the area of the biggest square is 1 square unit, then ½ of ½ of 1 = ¼ square unit. $s^2 = 1/4$; $s = 1/2$ unit.

52. **C**



The radius of the hollow portion is 2 units.
Thus, its volume = $\pi r^2 h = \pi(4)(3) = 12\pi$.
The radius of the whole cylinder is 3 units.
Thus, the volume of the whole cylinder is
$= \pi r^2 h = \pi(9)(3) = 27\pi$.
Thus, the volume of the concrete portion is $27\pi - 12\pi = 15\pi$.

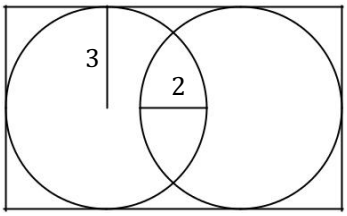
53. **B**
Let x be Jaz's age.
Her grandmother is $60 + x$.
Her mother is $3x - 3$.

$102 = x + (60 + x) + (3x - 3)$
$102 = 5x + 57$
$5x = 45$
$x = 9$

Thus, her mother is $3(9) - 3 = 24$ years old.

54. **C**
The side of square is equal to the diameter of the circle.
$s = 40/4 = 10$. Thus, the circumference of the circle
$= \pi d = 10\pi$.

55. **C**




The width of the figure is equal to 2 x radius of the circle = 2 x 3 = 6.
The length of the figure is 2 x diameter of the circles – 2 = 2 x 6 - 2 = 10.
Thus, the area of the rectangle is 6 x 10 = 60.

56. **B**
Joe is Jen's husband.

57. **C**
$0.028 - 3/125 = 0.028 - 0.024 = 0.004 = 1/125$

58. **E**
Let x be the number of tables with 4 chairs and y be the number of tables with 6 chairs.
$x + y = 20$ multiply by -6
$4x + 6y = 92$
eliminate y
$-6x - 6y = -120$
$4x + 6y = 92$
$-2x = -28$
$x = 14$

59. **E** Since it is beyond 75%, it must be 80%.
60. **B** The 2007 graph is half the 2008 graph, thus, 2:1.