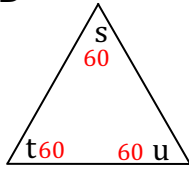


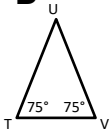
Math Practice Test 16
"More Practice" Answer Key

1. **D**



equilateral: all sides are equal
equiangular: all angles are equal
 The sum of the angles of a triangle is 180° .

2. **B**



Given the first measures of two of the interior angles of the triangle, we can say that the measure of the third angle is 30° , since the sum of the measures of the three interior angles in any given triangle is 180° .

a. **TU = UV;** (TRUE; Converse of Isosceles Triangle Theorem: If two angles of a triangle are equal in measure, then the sides opposite those angles are equal in measure)

b. **TV > UV;** (FALSE; If two angles of a triangle are not congruent, then the longer side is opposite the larger angle. Since $75^\circ > 30^\circ$, then both TU and UV are greater than TV.)

c. **TU > TV** (TRUE; same explanation as in b)

d. **$\angle U = 30^\circ$** (TRUE; Triangle Angle Sum Theorem: the sum of the measures of the three interior angles in any given triangle is 180° .)

3. **B**

$$2:3:5 = (2x):(3x):(5x)$$

$$2x + 3x + 5x = 180^\circ$$

$$10x = 180^\circ$$

$$x = 18^\circ$$

Largest Angle: $5x = 90^\circ$

4. **B**

A midpoint of a line segment is equidistant from the 2 end points.
 Distance $(-14, -6) = |-14 - (-6)|$
 $= |-8| = 8 - 6 + 8 = 2$

5. **B**

Since the first point is at $(0,0)$, and the midpoint is at $(4,2)$, this means that half of the line segment is 4 units to the right and 2 units upward. Thus, we need to extend it by another 4 units to the right and 2 units upward, getting $(8,4)$.

6. **C**

Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(21 - 5)^2 + (-9 - 3)^2}$$

$$d = \sqrt{(16)^2 + (-12)^2}$$

$$d = \sqrt{256 + 144}$$

$$d = \sqrt{400}$$

$$d = 20$$

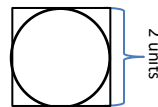
7. **C**

The side of square is equal to the diameter of the circle.

$$s = 40/4 = 10.$$

Thus, the circumference of the circle
 $= \pi d = 10\pi.$

8. **A**



diameter: 2 units

radius: $(\frac{1}{2})(\text{diameter}) = (\frac{1}{2})(2) = 1$ unit

Area: $\pi r^2 = \pi(1)^2 = \pi$

9. A

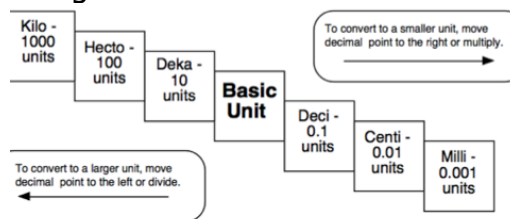
The Pythagorean Theorem ($a^2 + b^2 = c^2$) applies in any given right triangle. Thus, if the sides of the triangles are consecutive even integers, then we can substitute the lengths of the sides such that the resulting equation is

$$\begin{aligned} a^2 + (a + 2)^2 &= (a + 4)^2 \\ a^2 + (a^2 + 4a + 4) &= a^2 + 8a + 16 \\ a^2 + a^2 + 4a + 4 &= a^2 + 8a + 16 \\ 2a^2 + 4a + 4 &= a^2 + 8a + 16 \\ a^2 - 4a - 12 &= 0 \\ (a - 6)(a + 2) &= 0 \\ a &= 6, -2 \end{aligned}$$

Since the length of a side of a triangle cannot be negative, thus the length of the shortest side is 6.

10. A

Using Metric Conversion Ladder Method



<https://msgeshkesciencehub.wordpress.com/tag/metric-system/>

Using elimination of choices, D and E cannot be both correct since they have same unit, then $5 \text{ km} \neq 500 \text{ m}$

Math Practice Test 17
“More Practice” Answer Key

1. **D**

Let A, B and C be the any of sides of a triangle.

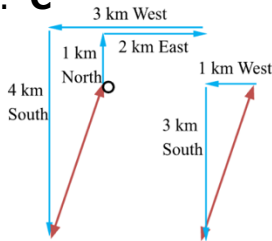
$A + B > C$; wherein A, B and C are the lengths of the three sides of a triangle. (Note: Values for A, B and C are interchangeable.)

$$10+9>8;$$

$$10+8>9;$$

$$9+8>10$$

2. **C**



Note: \circ origin; \rightarrow displacement

$$\text{Displacement} = \sqrt{3^2 + 1^2} = \sqrt{9 + 1} = \sqrt{10}$$

3. **C**

Use the Pythagorean theorem: $c^2 = a^2 + b^2$

$$12^2 = 5^2 + b^2$$

$$b^2 = 144 - 25 = 119$$

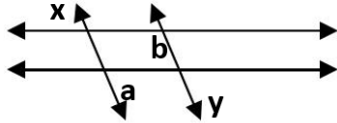
$$b = \sqrt{119}$$

4. **C**

Statement	Reason
1. $\overline{BD} = \overline{CD}$; $\overline{AD} = \overline{BD}$;	1. Definition of isosceles triangle
2. $\overline{AD} = \overline{CD}$;	2. Transitive Property of Equality
3. $m\angle DBC = m\angle DCB$; $m\angle DAB = m\angle DBA$; $m\angle DAC = m\angle DCA$	3. Isosceles Triangle Theorem
4. $m\angle DBC + m\angle DCB + 120^\circ = 180^\circ$	4. Definition of a triangle

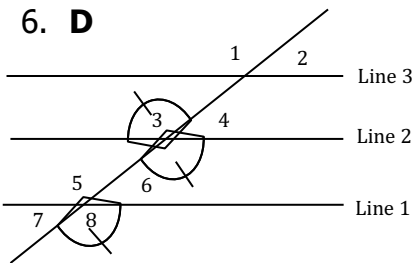
5. $m\angle DBC + m\angle DCB = 60^\circ$	5. Subtraction Property of Equality
6. $m\angle DBC + m\angle DCB = 2(m\angle DBC) = 60^\circ$	6. Addition Property of Equality
7. $m\angle DBC = 30^\circ$	7. Division Property of Equality
8. $m\angle DCB = 30^\circ$	8. Transitive Property
9. $m\angle DAB + m\angle DBA + m\angle DBC + m\angle DCB + m\angle DCA + m\angle DAC = 360^\circ$	9. Triangle Angle Sum Theorem
10. $m\angle DAB + m\angle DAB + m\angle DBC + m\angle DBC + m\angle DCA + m\angle DCA + m\angle DAC = 180^\circ$	10. Addition Property of Equality
11. $m\angle DAB + m\angle DBC + m\angle DAC = 90^\circ$	11. Division Property of Equality
12. $m\angle DAB + 30^\circ + m\angle DAC = 90^\circ$	12. Substitution
13. $m\angle DAB + m\angle DAC = 60^\circ$	13. Subtraction Property of Equality
14. $m\angle DAB + m\angle DAC = m\angle BAC = 60^\circ$	14. Angle Addition Postulate
15. $\angle BAC$ and $\angle x$ forms a linear pair and are supplementary	15. Definition of a Linear Pair; Linear Pair Theorem
16. $m\angle BAC + m\angle x = 180^\circ$	16. Definition of Supplementary
17. $60^\circ + m\angle x = 180^\circ$	17. Substitution of Values
18. $m\angle x = 120^\circ$	18. Subtraction Property of Equality

5. **A**



Statement	Reason
1. $m\angle X = m\angle A$	1. Alternate Exterior Angles
2. $m\angle A = m\angle B$	2. Alternate Interior Angle
3. $m\angle B = m\angle Y$	3. Vertical Angles
4. $m\angle X = m\angle Y$	4. Transitive Property of Equality

6. **D**



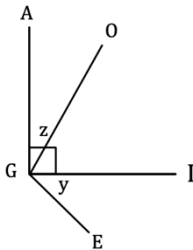
Line 3 and 8 are vertical angles.

7. **D**

$(16 \text{ in})(30 \text{ in}) = 480 \text{ in}^2$
Squares with side 1: 1 in by 1 in

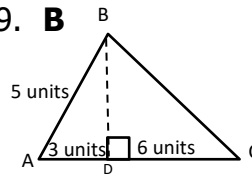
$$\begin{aligned} \text{Area}_{\text{square}} &: (1 \text{ in})(1 \text{ in}) = 1 \text{ in}^2 \\ \text{Area}_{\text{shaded}} &= \text{Area}_{\text{rectangles}} - \text{Total Area}_{\text{squares}} \\ &= 480 - (6)(1) \\ &= 480 - 6 = \mathbf{474 \text{ in}^2} \end{aligned}$$

8. **E**



$$\begin{aligned} \text{OGI} &= a \\ \angle \text{AGE} &= b \\ \angle z &= 90 - a \\ \angle y &= b - 90 \\ z - y &= (90 - a) - (b - 90) \\ z - y &= 90 - a - b + 90 \\ z - y &= \mathbf{180 - a - b} \end{aligned}$$

9. **B**



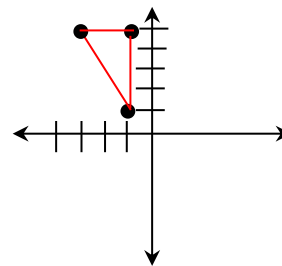
Since the dashed line is a perpendicular bisector, we can say that $\triangle ABD$ is a right triangle. Thus, the height is 4 units (Pythagorean triple: 3-4-5).

The area of the triangle can then be computed by adding the areas of the two right triangles.

$$\begin{aligned} A_{\text{total}} &= \frac{(AD)(DB)}{2} + \frac{(DC)(DB)}{2} = \frac{(3)(4)}{2} + \frac{(6)(4)}{2} \\ &= \frac{12}{2} + \frac{24}{2} = 6 + 12 = \mathbf{18 \text{ sq. units}} \end{aligned}$$

10. **C**

Plot the points $(-1,5)$, $(-1,1)$, and $(-3,5)$ on the cartesian plane as vertices of a triangle.

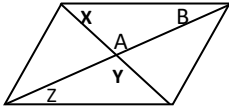


11. **C**

Use partitive proportion.
Let x be the original piece

$$\begin{aligned} 3 : 4 : 5 &= 12 \\ \therefore _ : _ : 2.5m &= x \\ \frac{2.5}{5} : \frac{x}{12} & \\ x &= 6 \end{aligned}$$

12. **A**



Statement	Reason
1. $\angle A$ and $\angle Y$ are vertical angles	1. Definition of Vertical Angles
2. $m\angle A = m\angle Y = 100^\circ$	2. Vertical Angle Theorem; Given
3. $m\angle A + m\angle B + m\angle X = 180$	3. Triangle Angle Sum Theorem
4. $100^\circ + m\angle B + 55^\circ = 180^\circ$	4. Given
5. $m\angle B = 25^\circ$	5. Subtraction Property of Equality
6. $m\angle B = m\angle Z$	6. Alternate Interior Angle Theorem
7. $m\angle Z = 25$	7. Transitive Property of Equality

13. **B**

a. A rectangle is always a square.

Rectangle: a quadrilateral with opposite sides parallel and 4 right angles.

Square: a quadrilateral with opposite sides parallel, 4 right angles and 4 equal sides.

*Not all rectangles have 4 equal sides. However, we can say that all squares are rectangles.

b. A square is always a rhombus.

Rhombus: a quadrilateral with opposite sides parallel and 4 equal sides.

*Since a square has parallel opposite sides and 4 equal sides, then we can say that this statement is true.

c. A rhombus is always a rhomboid.

Rhomboid: a quadrilateral with opposite sides parallel and opposite sides and angles equal.

*The adjacent sides of rhomboids may or may not be equal.

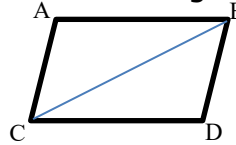
d. A rhomboid is always a rectangle.

*Even though opposite angles of rhomboids are equal, it is possible that these angles are not 90° .

14. **D**

The first three statements (Opposite angles are congruent, opposite sides are equal in length, and adjacent angles are always supplementary.) are among the properties of parallelograms.

Let ABCD be a parallelogram and BC be one of its diagonals.



Statement	Reason
1. $AB \parallel CD$; $AC \parallel BD$	Defn. of parallelogram
2. $\angle ACB = \angle DBC$; $\angle ABC = \angle DCB$	Alternate interior angles of parallel lines are congruent.
3. $\overline{BC} = \overline{BC}$	Reflexive Property
4. $\triangle ACB \cong \triangle DBC$	ASA Postulate
5. $\angle A = \angle D$;	Corresponding parts of congruent triangles are congruent
Statement 1 proved. You can also prove that $\angle B = \angle D$ by using the segment AD.	
6. $\overline{AB} = \overline{CD}$; $\overline{AC} = \overline{BD}$	Corresponding parts of congruent triangles are congruent
Statement 2 proved.	
7. $m\angle A + m\angle ACB + m\angle ABC = 180$	Definition of a triangle
8. $m\angle ABC \cong m\angle DCB$	Definition of congruent angles
9. $m\angle A + m\angle ACB + m\angle DCB = 180$	Addition Property of Equality
10. $m\angle ACB + m\angle DCB = m\angle C$	Angle Addition Postulate
11. $m\angle A + m\angle C = 180$	Addition Property of Equality
Statement 3 proved. You can also prove that $m\angle B + m\angle D = 180$ if the diagonal used is AD.	

15. **D**

$$3:4:5 = (3x):(4x):(5x)$$

$$3x + 4x + 5x = 180^\circ$$

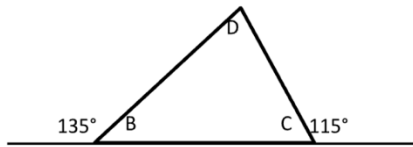
$$12x = 180^\circ$$

$$x = 15$$

$$\text{Largest angle: } 5x = 75^\circ$$

Math Practice Test 18
"More Practice" Answer Key

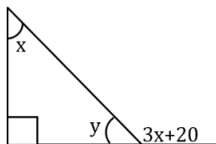
1. **C**



Let B and C be the other angles in the triangle.

Statement	Reason
1. 135° and $\angle B$ forms a linear pair; 115° and $\angle C$ forms a linear pair	1. Definition of a linear pair
2. 135° and $\angle B$ are supplementary; 115° and $\angle C$ are supplementary	2. Linear Pair Theorem
3. $135^\circ + m\angle B = 180^\circ$; $115^\circ + m\angle C = 180^\circ$	3. Definition of Supplementary angles
4. $m\angle B = 45^\circ$; $m\angle C = 65^\circ$	4. Subtraction Property of Equality
5. $m\angle B + m\angle C + m\angle D = 180^\circ$	5. Triangle angle sum theorem
6. $45^\circ + 65^\circ + m\angle D = 180^\circ$	6. Addition Property of Equality
7. $m\angle D = 70^\circ$	7. Subtraction Property of Equality

2. **C**



(Sum of remote interior angles = exterior angle)

$$3x + 20 = 90 + x$$

$$2x = 70$$

$$x = 35$$

(Sum of interior angles of a triangle = 180)

$$90 + x + y = 180$$

$$y = 180 - 90 - 35$$

$$y = 55$$

3. **B**

$$SA_{\text{cube}} = 6s^2$$

$$216 \text{ cm}^2 = 6s^2$$

$$s^2 = 36 \text{ cm}^2$$

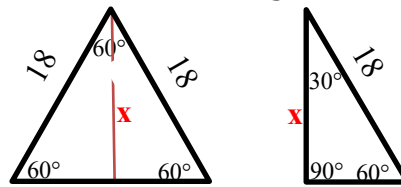
$$s = 6 \text{ cm}$$

$$V_{\text{cube}} = s^3 = (6 \text{ cm})^3 = 216 \text{ cm}^3$$

4. **B**

Since the triangle is equilateral, we can also say that the triangle is equiangular, with each angle = 60° .

If the perimeter of an equilateral triangle is 54, then the length of a side is $\frac{54}{3}$ or 18.



$$18 \quad \frac{18}{2} \text{ or } 9$$

Pythagorean Theorem: $a^2 + b^2 = c^2$

$$x^2 + 9^2 = 18^2$$

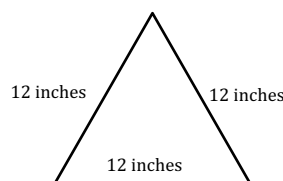
$$x^2 + 81 = 324$$

$$x^2 = 324 - 81 = 243$$

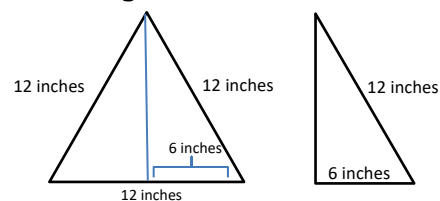
$$x = \sqrt{243} = \sqrt{3^5} = 9\sqrt{3}$$

5. **C**

If the perimeter of an equilateral triangle is 36 inches, then each side measures $36/3$ or 12 inches.



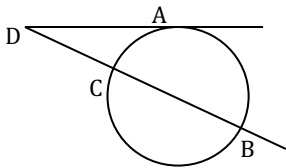
To measure the height, we can draw a perpendicular bisector in the triangle and consider the height as one of the sides of the half-triangle.



We can use the Pythagorean Theorem to look for the measurement of the height.

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 (6 \text{ inches})^2 + b^2 &= (12 \text{ inches})^2 \\
 36 \text{ inches}^2 + b^2 &= 144 \text{ inches}^2 \\
 b^2 &= 144 \text{ inches}^2 - 36 \text{ inches}^2 \\
 b^2 &= 108 \text{ inches}^2 \\
 b &= 6\sqrt{3} = \text{height} \\
 A_{\text{triangle}} &= \frac{1}{2}bh = \frac{1}{2}(12)(6\sqrt{3}) \\
 &= (6)(6\sqrt{3}) = 36\sqrt{3}
 \end{aligned}$$

6. **D**

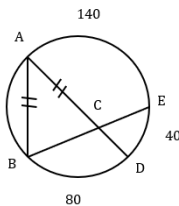


To get the measurement of \widehat{AC} ,

$$\begin{aligned}
 \angle ADB &= \frac{1}{2}(\widehat{AB} - \widehat{AC}) \\
 2 \left[20 &= \frac{1}{2}(160 - \widehat{AC}) \right] 2 \\
 40 &= 160 - \widehat{AC} \\
 \widehat{AC} &= 160 - 40 \\
 \widehat{AC} &= 120^\circ
 \end{aligned}$$

The measurement of \widehat{AC} is 120° .

7. **A**



$$\begin{aligned}
 AB &= AC \\
 \widehat{AE} &= 140^\circ \\
 \widehat{ED} &= 40^\circ \\
 \angle B &= \frac{1}{2}m\widehat{AE} \\
 \angle B &= \frac{1}{2}(140^\circ) \\
 \angle B &= 70^\circ
 \end{aligned}$$

Since $AB = AC$, $\triangle ABC$ is an isosceles triangle. The bases of $\triangle ABC$ are also equal.

Since the sum of the angles of a triangle is 180° ,

$$\begin{aligned}
 \angle A &= 180 - 70 - 70 = 40^\circ \\
 \widehat{BD} &= 2 \times m\angle A \\
 \widehat{BD} &= 2 \times 40^\circ \\
 \widehat{BD} &= 80^\circ
 \end{aligned}$$

One complete rotation of a circle is 360° .
To get the measurement of $\widehat{AB} = 360 - m\widehat{AE} - m\widehat{ED} - m\widehat{BD} = 360 - 140 - 40 - 80 = 40^\circ$
 $\widehat{AB} = 40^\circ$

8. **A**

$$\begin{aligned}
 12+6 &= 18; \text{ height of bigger triangle} \\
 12:20 &:: 18:20 + x \\
 20 + x &= \frac{(20)(18)}{12} = 30 \\
 x &= 30 - 20 = 10
 \end{aligned}$$

9. **A**

Using corresponding sides of similar triangles are proportional then,

$$\begin{aligned}
 \frac{x+2}{6} &= \frac{2}{3} \\
 3x+6 &= 12 \\
 x &= 2
 \end{aligned}$$

10. **B**

Let h be the heights and s be the lengths of the shadow. The ratio of height and length of the tree is equal to the ratio of the height and length of the stick.

$$\begin{aligned}
 \frac{h}{s} &= \frac{1 \text{ m}}{3 \text{ m}} = \frac{x}{15.3 \text{ m}} \\
 x &= 5.1 \text{ m}
 \end{aligned}$$

11. **B**

Ratio and Proportion

$$\begin{aligned}
 \frac{x}{8} &= \frac{6}{9} \\
 x &= \frac{48}{9} = 5\frac{1}{3} = 5'4''
 \end{aligned}$$

12. **B**

$$\begin{aligned}
 15:20 &:: 6:x \\
 x &= \frac{(20)(6)}{15} = \frac{120}{15} = 8
 \end{aligned}$$

Math Practice Test 19
"More Practice" Answer Key

1. **B**

Let r be the radius of the rear wheel and f be the radius of the front wheel. The relationship between the two radius is: $r = 2f$.

Getting the circumference of the rear wheel:

$$C_{\text{rear}} = \pi d = \pi \times 2r$$

Substituting the relationship of the two wheels into the equation above,

$$\pi d = \pi \times 2r = \pi \times 2(2f) = 4\pi f$$

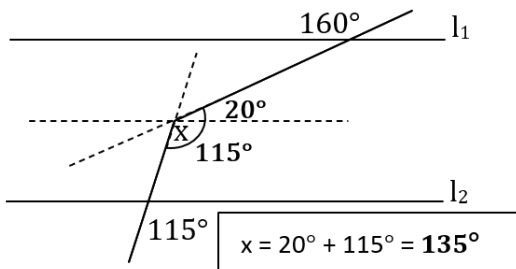
Thus,

$$C_{\text{front}} = \pi d = \pi \times 2f = 2\pi f$$

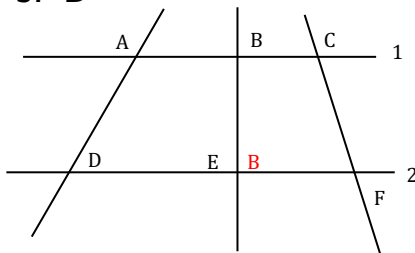
$$C_{\text{rear}} = 2C_{\text{front}}$$

2. **E**

Use properties of transversal.



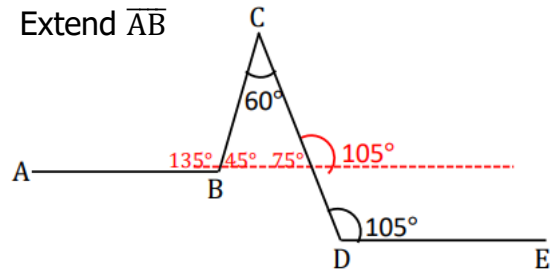
3. **D**



- a) $\angle A \cong \angle E$ FALSE
- b) $\angle C \cong \angle F$ FALSE
- c) $m\angle D + m\angle E = 90^\circ$ FALSE
- d) $m\angle B + \angle E = 180^\circ$ **TRUE**
 $\angle B$ and $\angle E$ are supplementary angles
- e) $m\angle D + m\angle F = 180^\circ$ FALSE

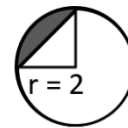
4. **C**

Extend \overline{AB}



The measurement of $\angle ABC$ is 135° .

5. **A**



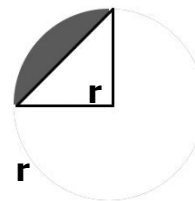
$$A_{\text{circle}} = \pi r^2 = \pi(2)^2 = 4\pi$$

$$A_{\text{quarter-circle}} = (1/4)(A_{\text{circle}}) = (1/4)(4\pi) = \pi$$

$$A_{\text{triangle}} = 1/2 b \cdot h = 1/2 (2)(2) = 1/2 (4) = 2$$

$$A_{\text{shaded}} = A_{\text{quarter-circle}} - A_{\text{triangle}} = \pi - 2$$

6. **B**



The area of the triangle is $\frac{r^2}{2}$.

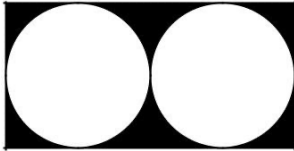
The area of the quarter circle is $\frac{\pi r^2}{4}$.

Subtracting the area of the triangle from the area of the quarter circle, we get

$$\frac{\pi r^2}{4} - \frac{r^2}{2} = \frac{22}{7}(r^2) - \frac{r^2}{2}$$

$$= \frac{22r^2}{28} - \frac{14r^2}{28} = \frac{8r^2}{28} = \frac{2}{7}r^2$$

7. **D**



The area of the circle $9\pi = \pi r^2$

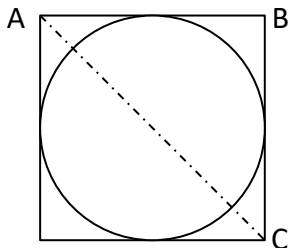
Thus, the radius of each circle is 3, and the diameter $= 2r = 6$. With two circles side by side, the length of the rectangle $= 2d = 12$ and the width of the rectangle is $d = 6$.

The area of the whole rectangle is $12 \times 6 = 72$ square units.

The area of the shaded region $= 72 - 2(9\pi) = 72 - 18\pi = \mathbf{18(4 - \pi)}$.

8. **A**

This figure is easier to solve if we make it to of it and combine to form a square since the triangle is an isosceles right triangle (half of a square).



Thus, we can say that the area inside the triangle but outside the semi-circle in the original figure is half the area of the difference of the square having a diagonal of $4\sqrt{2}$ and a circle having a diameter of 4 (4 is the side of the square having a diagonal of $4\sqrt{2}$; it is a property of a square that the diagonal is $s\sqrt{2}$).

$$A = \frac{1}{2}(A_{\text{square}} - A_{\text{circle}})$$

$$A = \frac{1}{2}(s^2 - \pi r^2)$$

$$A = \frac{1}{2}(4^2 - (\pi)2^2)$$

$$A = \frac{1}{2}(16 - 4\pi)$$

$$A = \mathbf{8 - 2\pi}$$

9. **B**

$$SA_{\text{cylinder}} = 4\pi r^2$$

$$256\pi \text{mm}^2 = 4\pi r^2$$

$$64 \text{mm}^2 = r^2$$

$$r = \mathbf{8 \text{ mm}}$$

Since the radius of the ball is 8 mm, the minimum radius of a cylinder for a ball to get through it is also 8 mm.

10. **B**

The volume of the prism is equal to $V = Ah$ where A is the area of the base. In this case, a prism with a square base has area $V = s^2h$.

$$54 = s^2 \times 6$$

$$s^2 = 9$$

$$s = \mathbf{3}$$

11. **C**



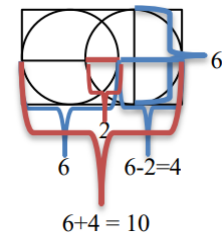
The radius of the hollow portion is 2 units.

Thus, its volume $= \pi r^2 h = \pi(4)(3) = 12\pi$.

The radius of the whole cylinder is 3 units. Thus, the volume of the whole cylinder is $= \pi r^2 h = \pi(9)(3) = 27\pi$.

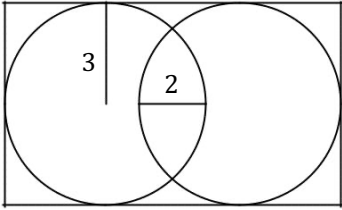
Thus, the volume of the concrete portion is $27\pi - 12\pi = \mathbf{15\pi}$.

12. **B**



Area $= (\text{length})(\text{width}) = (10)(6) = \mathbf{60 \text{ sq. units}}$

13. C

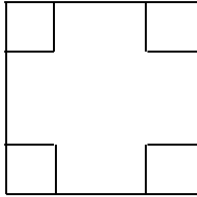


The width of the figure is equal to 2 x radius of the circle = 2 x 3 = 6.

The length of the figure is 2 x diameter of the circles - 2 = 2 x 6 - 2 = 10.

Thus, the area of the rectangle is 6 x 10 = **60**.

14. D



The area of the large square is 16. That means, the side of the large square is equal to $\sqrt{16} = 4$. The perimeter of each small square is equal to 4. Thus, the side of each small square is $4/4 = 1$. The area of each small square is

$1 \times 1 = 1$ square unit. 4 small squares = 4 square units.

$16 - 4 = \mathbf{12}$ square units.

15. C

(Using Midpoint Theorem)

$$\overline{AK} = \overline{AT} = \overline{TK} = \frac{24}{3} = 8$$

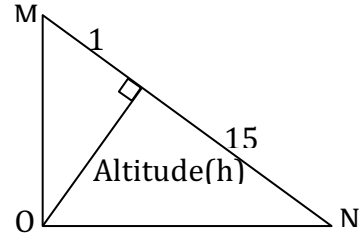
$$\overline{AK} \parallel \overline{ED} \text{ and } \overline{ED} = \frac{1}{2}\overline{AK} \text{ then } \overline{ED} = 4$$

$$\overline{AT} \parallel \overline{FD} \text{ and } \overline{FD} = \frac{1}{2}\overline{AT} \text{ then } \overline{FD} = 4$$

$$\overline{TK} \parallel \overline{EF} \text{ and } \overline{EF} = \frac{1}{2}\overline{TK} \text{ then } \overline{EF} = 4$$

$$\text{Perimeter}_{\text{triangle DEF}} = 4 + 4 + 4 = \mathbf{12}$$

16. D



Using similar triangles.

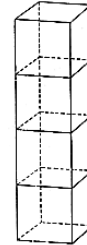
$$\frac{h}{10} = \frac{15}{h}$$

$$h^2 = 150$$

$$h = \sqrt{150}$$

$$h = 5\sqrt{6}$$

17. C



$$SA = 6s^2 = 54m^2$$

$$s^2 = 9m^2$$

$$SA_{\text{rectangular prism}} = 18 \times s^2$$

$$SA = 18 \times 9m^2 = 162m^2$$

Math Practice Test 20
“More Practice” Answer Key

1. **A**

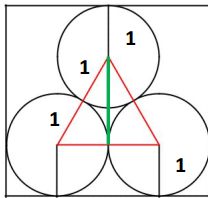
To get the number of posts needed, divide 114m by 6cm,

$$\frac{114}{6} = 19$$

Subtract 1 from 19 because 1st post and last post are not connected.

$$19 - 1 = \mathbf{18 \text{ posts}}$$

2. **E**



The red triangle inside is equilateral triangle with side 2 units. The angles inside the equilateral triangle are equal to 60°. To get the height of the triangle (the green line), we use the 30-60-90 triangle relationship.

If the hypotenuse is 2 units, the side opposite to the 60°, which happens to be the height of the equilateral triangle, is equal to $\sqrt{3}$. The total height of the figure is $2 + \sqrt{3}$.

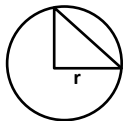
3. **B**

Area of triangle: 4cm^2 ;

Side of square: $\sqrt{8} =$ Radius of circle

Area of circle: $\pi r^2 = \pi(\sqrt{8})^2 = \mathbf{8\pi \text{ cm}^2}$

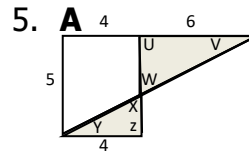
4. **C**



If the area of the circle is 64π square units, then the radius of the circle is

$$\begin{aligned} A &= \pi r^2 \\ 64\pi &= \pi r^2 \\ 64 &= r^2 \\ \mathbf{r} &= \mathbf{8 \text{ units}} \end{aligned}$$

Since the radius of the circle is also the base and height of the triangle, the area of the triangle is $\frac{(b)(h)}{2} = \frac{(r)(r)}{2} = \frac{(8)(8)}{2} = \frac{64}{2} = \mathbf{32 \text{ sq. units}}$



Statement	Reason
1. $\angle W$ & $\angle X$ are vertical angles	1. Definition of Vertical Angles
2. $m\angle W = m\angle X$	2. Vertical Angle Theorem
3. $\angle V$ and $\angle Y$ are alternate interior angles	3. Definition of Alternate Interior Angles
4. $m\angle V = m\angle Y$	4. Alternate Interior Angle Theorem
5. ΔWUV is similar to ΔXZY	5. AA Similarity Postulate
6. The sides of ΔWUV are in proportion to ΔXZY	6. Definition of Similar Triangles

$$WU:UV::XZ:ZY$$

$$\text{Note: } WU + XZ = 5$$

$$\text{Let } x = WU$$

$$x + XZ = 5$$

$$XZ = 5 - x$$

$$x:6 :: (5-x):4$$

$$(x)(4) = (6)(5-x)$$

$$4x = 30 - 6x$$

$$10x = 30$$

$$x = 3 = WU$$

$$5 - x = 5 - 3 = 2 = XZ$$

$$\text{Area}_{\text{triangle}} = \frac{1}{2}bh$$

$$\text{Area}_{\Delta WUV} = \left(\frac{1}{2}\right)(6)(3) = \frac{1}{2}18 = 9$$

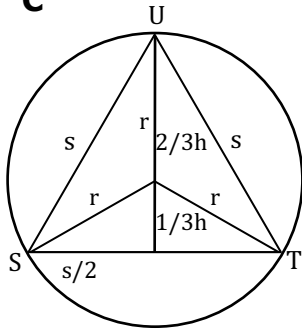
$$\text{Area}_{\Delta XZY} = \left(\frac{1}{2}\right)(4)(2) = \left(\frac{1}{2}\right)8 = 4$$

$$\text{Area}_{\text{shaded}} = 9 + 4 = \mathbf{13}$$

6. **D**

Area_{shaded}:Area_{ABCD}
 13:(5)(4); **13: 20**

7. **C**



Since triangle UST is equilateral, then
 $s = 2$

$h = 6\sqrt{3}$ (using special right triangle)
 radius = (r) = $\frac{2}{3}h$ (using median and perpendicular bisector)

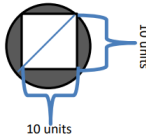
$$r = \left(\frac{2}{3}\right)(6\sqrt{3}) = 4\sqrt{3}$$

Area_{shaded} = Area_{circle} - Area_{triangle}

$$\text{Area}_{\text{shaded}} = \pi(4\sqrt{3})^2 - \frac{12(6\sqrt{3})}{2}$$

$$\text{Area}_{\text{shaded}} = 48\pi - 36\sqrt{3}$$

8. **B**



If the perimeter of the square is 40, then each side is 40/4 or 10 units long and its area is 100 square units. If you draw a diagonal inside the inscribed square, you can notice that this line is also the diameter of the circle. To compute for the length of this line, we can use the Pythagorean Theorem. Length of Diagonal:

$$x^2 + y^2 = z^2$$

$$10^2 + 10^2 = z^2$$

$$200 = z^2$$

$$z = 10\sqrt{2}$$

$$\text{diagonal/diameter} = 10\sqrt{2}$$

It follows that the radius of the circle is $\frac{10\sqrt{2}}{2}$ or $5\sqrt{2}$. Thus the area of the circle is:

$$A = \pi r^2 = \pi(5\sqrt{2})^2 = 50\pi.$$

$$A_{\text{shaded}} = A_{\text{circle}} - A_{\text{square}} = 50\pi - 100$$

9. **C**

Width of smallest triangle: $2x$;
 Width of new triangle: $8x$;
 Height of smallest triangle: y ;
 Height of new triangle: $4y$;
 Area of smallest triangle: $\frac{(2x)(y)}{2} = xy$;
 Area of new triangle: $\frac{(8x)(4y)}{2} = 16xy$;
 Area is increased **16 times**

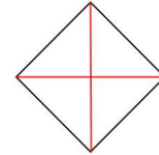
10. **A**

$$\text{Area}_{\text{circle}} = \pi r^2 = 64\pi$$

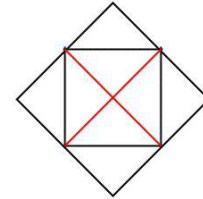
$$r = 8 = \text{diagonal of square}(d)$$

$$\text{Area}_{\text{square}} = \frac{d^2}{2} = \frac{8^2}{2} = 32$$

11. **B**



The smaller square is half the area of the biggest square.



The smallest square is half the area of the smaller square.

If the area of the biggest square is 1 square unit, then $\frac{1}{2}$ of $\frac{1}{2}$ of 1 = $\frac{1}{4}$ square unit. $s^2 = 1/4$; $s = \mathbf{1/2 \text{ unit}}$.

12. **C**

Volume of cylinder: $\pi r^2 h$;

Since π and height are constant, ratio of volume depends on r^2 .

$$\text{Ratio: } 1^2:2^2:4^2 = \mathbf{1:4:16}$$

13. **A**

$A_{\text{smaller circle}} : A_{\text{bigger circle}}$

$$\pi r_{\text{small}}^2 : \pi r_{\text{big}}^2$$

$$d_{\text{small}} = \text{radius}_{\text{big}}$$

$$2 \text{radius}_{\text{small}} = \text{radius}_{\text{big}}$$

$$A_{\text{small}} : A_{\text{big}}$$

$$\begin{aligned} \pi(\text{radius}_{\text{small}})^2 &: \pi(\text{radius}_{\text{big}})^2 \\ \pi(\text{radius}_{\text{small}})^2 &: \pi(2\text{radius}_{\text{small}})^2 \\ \pi(\text{radius}_{\text{small}})^2 &: \pi(4)(\text{radius}_{\text{small}})^2 \\ &\mathbf{1:4} \end{aligned}$$

14. C

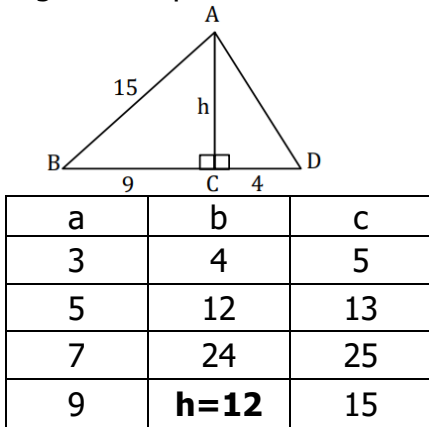
Let R be the radius of the bigger circle, r be the radius of the smaller circle.

$$R = 3r$$

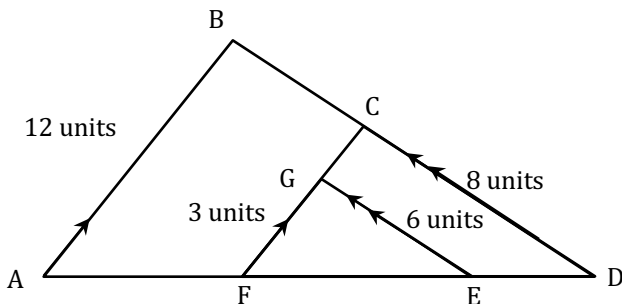
If the circumference of the smaller circle is $2\pi r = 6\pi$, then the radius of the smaller circle is 3. Thus, the radius of the bigger circle is $R = 3(3) = 9$. Therefore, the circumference of the bigger circle is $2\pi R = 2\pi(9) = \mathbf{18\pi}$.

15. B

Use pythagorean triples.



16. E



Let us look at ΔFCD and ΔFGE first since they are similar triangles. Using RAP, we can say that:

$$\frac{FC}{FG} = \frac{CD}{GE}$$

$$\begin{aligned} \frac{FC}{3} &= \frac{8}{6} \\ FC &= 4 \end{aligned}$$

Since we now know the measure of FC, we now find the measure of BD so that we can solve BC by subtracting the measure of CD from BD. We will use the ΔABD and ΔFCD since they are similar triangles as well.

$$\begin{aligned} \frac{BD}{CD} &= \frac{AB}{FC} \\ \frac{BD}{8} &= \frac{12}{4} \\ BD &= 24 \end{aligned}$$

Therefore, $BC = BD - CD = 24 - 8 = \mathbf{16 \text{ units}}$

17. B

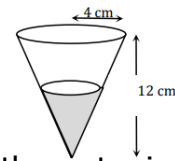
$$A_{\text{square}} = s^2 = 36\text{cm}^2$$

$$s = 6\text{cm}$$

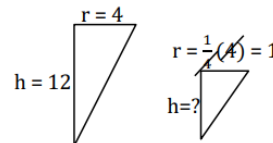
$$\text{Perimeter}_{\text{square}} = 4s = 4(6) = 24 \text{ cm}$$

$$\text{Perimeter}_{\text{square}} = \text{Perimeter}_{\text{triangle}} = \mathbf{24 \text{ cm}}$$

18. A



The radius of the water is $\frac{1}{4}$ of that of the cone.



To get the height of the water, use Ratio and Proportion (RAP)

$$\begin{aligned} \frac{4}{12} &= \frac{1}{h} \\ \frac{4h}{4} &= \frac{12}{4} \\ h &= 3 \end{aligned}$$

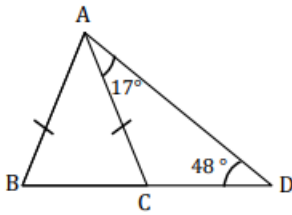
The height of the water is 3. Now use the formula to get the volume of a cone to get the volume of the water.

$$\begin{aligned} r &= 1 \\ h &= 3 \\ V_{\text{cone}} &= \frac{\pi r^2 h}{3} \\ V_{\text{cone}} &= \frac{\pi(1)^2(3)}{3} \\ V &= \pi\text{cm}^3 \end{aligned}$$

The volume of the water inside the cone is πcm^3 .

Math Practice Test 21
"More Practice" Answer Key

1. **A**



$$\begin{aligned} m\angle ACB &= m\angle CAD + m\angle ADC \\ m\angle ACB &= 17^\circ + 48^\circ = 65^\circ \\ m\angle ACB &\cong m\angle ABC \\ m\angle BAC &= 180^\circ - (m\angle ACB + m\angle ABC) \\ m\angle BAC &= 180^\circ - 130^\circ = 50^\circ \end{aligned}$$

2. **B**

Let w = width, l = length, P = perimeter

$$\begin{aligned} w &= \frac{l}{2} + 2 \\ l &= w + 3 \end{aligned}$$

we know that $P = 2w + 2l$
 Using the equation of w and P ,
 $P = 2\left(\frac{l}{2} + 2\right) + 2l$
 $P = l + 4 + 3l$
 $P = \mathbf{4 + 3l}$

3. **E**

Let w = width, l = length, P = perimeter
 $w = -\frac{1}{2}l - 2$.

we know that $P = 2w + 2l$
 $40 = 2w + 2l$
 $40 = 2\left(-\frac{1}{2}l - 2\right) + 2l$
 $40 = l - 4 + 2l$
 $40 = 3l - 4$
 $44 = 3l$
 $l = \mathbf{44/3}$

4. **C**

Let x be the width of the rectangle
 $x+3$ be the length of the rectangle
 $2(x+3) + 2(x) = 34$
 $2x + 6 + 2x = 34$
 $4x + 6 = 34$
 $4x = 34 - 6 = 28$

$$\begin{aligned} x &= \frac{28}{4} = 7 \\ x + 3 &= 10 \\ \text{Area} &= (\text{length})(\text{width}) = (10)(7) \\ &= \mathbf{70 \text{ sq. units}} \end{aligned}$$

5. **B**

Let w = width
 length = $2w$

$$\begin{aligned} \text{Area} &= (2w)(w) = 2w^2 \\ \text{Perimeter} &= 2(l+w) = 2(2w+w) = 6w \end{aligned}$$

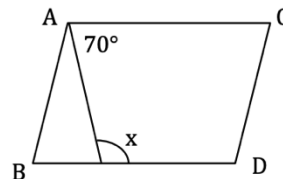
Given: Area = Perimeter + 8

Substitute values:
 $2w^2 = 6w + 8$
 $2w^2 - 6w - 8 = 0$
 $w^2 - 3w - 4 = 0$
 $(w-4)(w+1) = 0$

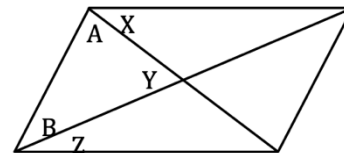
Since width must be positive, $w=4$. Since it is given that the length is twice its length, the width is 8.

6. **D**

Given $AC \parallel BD$, then $x + 70^\circ = 180$ (the property of transversal), $x=110^\circ$

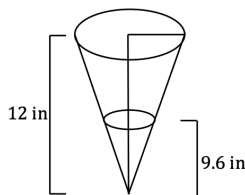


7. **E**

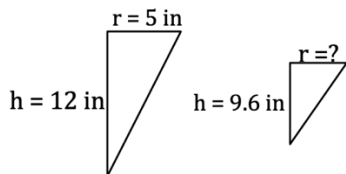


Statement	Reason
1. $m\angle A + m\angle B + m\angle Y = 180$	1. Triangle Angle Sum Theorem
2. $m\angle Z + m\angle B + m\angle A + m\angle X = 180$	2. Consecutive Angles of a Parallelogram
3. $m\angle A + m\angle B + m\angle Y = m\angle Z + m\angle B + m\angle A + m\angle X$	3. Transitive Property of Equality
4. $m\angle Y = m\angle Z + m\angle X$	4. Subtraction Property of Equality
5. $m\angle Z = -m\angle X + m\angle Y$	5. Subtraction Property of Equality

8. B



To get the radius of the water, use the Ratio and Proportion (RAP)



$$\frac{5}{12} = \frac{r}{9.6}$$

$$\frac{48}{12} = \frac{12r}{12}$$

$$r = 4$$

To get the area of the circular surface of the water, use the formula to get the area of circle where $r = 4$.

$$A(\text{circle}) = \pi r^2$$

$$A(\text{circle}) = \pi(4)^2$$

$$A(\text{circle}) = 16\pi \text{ in}^2$$

The area of the circular surface of the water is $16\pi \text{ in}^2$.

9. D

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

$$245\pi \text{ cm}^3 = \frac{1}{3}\pi r^2 (15 \text{ cm})$$

$$735\pi \text{ cm}^3 = \pi r^2 (15 \text{ cm})$$

$$49\pi \text{ cm}^2 = \pi r^2 = \text{Area of circular base}$$

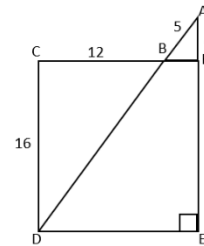
10. D

There was initially $\frac{1}{2} V$ of water.

$\frac{1}{6} V$ remained after 120 mL has been removed. Thus, $\frac{1}{2} V - \frac{1}{6} V = 120 \text{ mL}$
 $\frac{1}{3} V = 120 \text{ mL}$

$$V = 360 \text{ mL.}$$

11. D



Statement	Reason
1. $DE \parallel BF$	1. A trapezoid has one pair of parallel sides
2. $m\angle FED = m\angle AFB = 90^\circ$	2. Corresponding Angles Postulate
3. $\triangle FAB$ is a right triangle	3. Definition of a right triangle
4. $BF = 3$	4. Pythagorean Theorem (3-4-5 Pythagorean Triple)
5. $CF = CB + BF$	5. Segment Addition Postulate
6. $CF = 12 + 3 = 15$	6. Substitution of Values; Given
Since CDEF is a rectangle, then $CF = DE$.	

$$\begin{aligned} \text{Area}_{\text{trapezoid}} &= \frac{b_1 + b_2}{2} h = \frac{3 + 15}{2} 16 = \frac{18}{2} 16 \\ &= (9)(16) = 144 \text{ sq. units} \end{aligned}$$

12. C

The hypotenuse of triangle ABC is equal to $2\sqrt{2}$. If the ratio of the hypotenuse of triangle DEF to triangle ABC is $2:2\sqrt{2}$ which can be simplified to $1:\sqrt{2} = \sqrt{2}:2$. Since $BC = 2$, the length of EF is equal to $\sqrt{2}$.

13. C

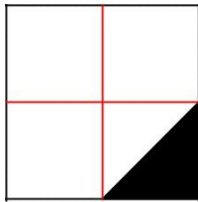
$$\text{Area}_{\Delta FOX} = 160 = \frac{bh}{2}$$

$$160 = \frac{(20)h}{2}$$

$$h = 16$$

$$\text{Area}_{\Delta FOX} = \frac{(5)(16)}{2} = 40$$

14. A



The perimeter of the square is 16. Thus, its side = $16/4 = 4$. Half its side is the side of the triangle. From the illustration, we can see that the triangle is $1/2$ of $1/4$ of the area of the whole square. Since its side is 4, the area of the square = $s \times s = 4 \times 4 = 16$. $1/2 \times 1/4 \times 16 = 2$.

Or, since we know that half the side of the square is the side of the triangle, the area of the triangle is $bh/2 = (2 \times 2)/2 = 2$

15. C



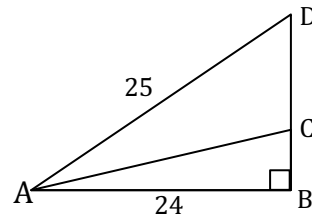
The diagonal of the square = radius of the quarter circle = 6.

$$\text{The area of the quarter circle} = \frac{\pi r^2}{4} = \frac{\pi \times 6^2}{4} = 9\pi$$

The area of the square is $d^2/2 = 6^2/2 = 18$. Thus, the area of the shaded region is $9\pi - 18$.

16. B

$\overline{DB} = 7$ (pythagorean triples)



$$\text{Area}_{ACD} = \text{Area}_{ABD} - \text{Area}_{ABC}$$

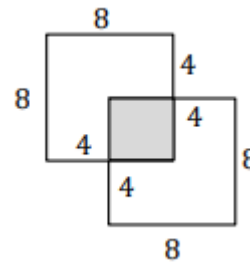
$$\text{Area}_{ACD} = \frac{24(7)}{2} - 25 = 59$$

17. E

If you are given an equilateral triangle circumscribed in a circle, there is a formula $r = \frac{s}{\sqrt{3}} P$ (wherein r is the radius of a circle and s is the side of the equilateral triangle).

Given R as the radius of the circle, we can say that $s = R\sqrt{3}$. Thus the perimeter $P = 3s = 3(R\sqrt{3}) = 3\sqrt{3} R$.

18. C

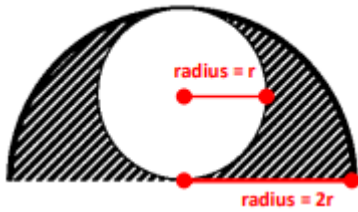


$$\text{Area}_{\text{square}} = s^2 = 16 \text{ m}^2$$

$$s = 4 \text{ m}$$

$$\text{Perimeter}_{\text{whole figure}} = 8(4) + 4(4) = 48 \text{ m}$$

19. A



$$\text{Area}_{\text{Shaded Region}} = \text{Area}_{\text{Semicircle}} - \text{Area}_{\text{Circle}}$$

$$\text{Area}_{\text{Shaded Region}} = \frac{\pi r^2}{2} - \pi r^2$$

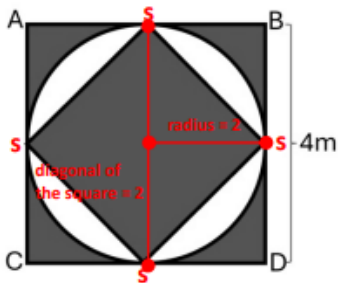
$$\text{Area}_{\text{Shaded Region}} = \frac{\pi(2r)^2}{2} - \pi r^2$$

$$\text{Area}_{\text{Shaded Region}} = \frac{4\pi r^2}{2} - \pi r^2$$

$$\text{Area}_{\text{Shaded Region}} = 2\pi r^2 - \pi r^2$$

$$\text{Area}_{\text{Shaded Region}} = \pi r^2 \text{ sq. units}$$

20. E



$$\text{Area}_{\text{Shaded Region}} = (\text{Area}_{\text{Bigger Square}} - \text{Area}_{\text{Circle}}) + \text{Area}_{\text{Smaller Square}}$$

$$\text{Area}_{\text{Shaded Region}} = (s^2 - \pi r^2) + \frac{\text{diagonal}^2}{2}$$

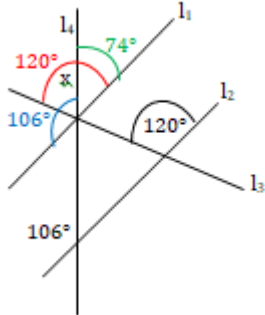
$$\text{Area}_{\text{Shaded Region}} = (4^2 - \pi(2)^2) + \frac{4^2}{2}$$

$$\text{Area}_{\text{Shaded Region}} = 16 - 4\pi + 8$$

$$\text{Area}_{\text{Shaded Region}} = 24 - 4\pi \text{ m}^2$$

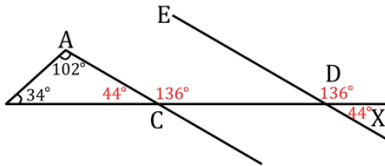
Math Practice Test 22
"More Practice" Answer Key

1. **A**



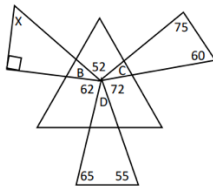
$$\angle x = 120 - 74 = 46^\circ$$

2. **A**



Angle X is equal to 44° .

3. **B**

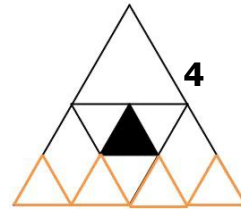


Note: \square right angle = 90°

Statement	Reason
1. $75^\circ + 60^\circ + m\angle C = 180^\circ$	1. Triangle Angle Sum Theorem
2. $m\angle C = 45^\circ$	2. Subtraction Property of Equality
3. $65^\circ + 55^\circ + m\angle D = 180^\circ$	3. Triangle Angle Sum Theorem
4. $m\angle D = 60^\circ$	4. Subtraction Property of Equality
5. $52^\circ + m\angle C + 72^\circ + m\angle D + 62^\circ + m\angle B = 360^\circ$;	5. The sum of all angles that meet at a point is equal to 360° .

6. $52^\circ + 45^\circ + 72^\circ + 60^\circ + 62^\circ + m\angle B = 360^\circ$	6. Substitution of Values
7. $m\angle B = 69^\circ$	7. Subtraction Property of Equality
8. $90^\circ + m\angle X + m\angle B = 360^\circ$;	8. Triangle Angle Sum Theorem
9. $90^\circ + m\angle X + 69^\circ = 360^\circ$;	9. Substitution of Values
10. $m\angle X = 21^\circ$	10. Subtraction Property of Equality

4. **D**

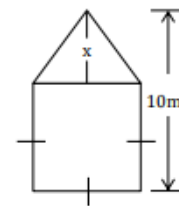


It can be seen that the length of the shaded triangle is 1.

Thus, its height is equal to $\frac{\sqrt{3}}{2}$. Solving $bh/2$,

$$\frac{1 \times \frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{3}}{4}$$

5. **A**



$$\text{Area}_{\text{square}} + \text{Area}_{\text{triangle}} = 48\text{m}^2$$

$$s^2 + \frac{bh}{2} = 48\text{m}^2$$

$$(10 - x)^2 + \frac{(10 - x)(x)}{2} = 48$$

$$\cancel{2} \left[100 - 20x + x^2 + \left(\frac{10x - x^2}{\cancel{2}} \right) = 48 \right] 2$$

$$200 - 40x + 2x^2 + 10x - x^2 = 96$$

$$2x^2 - x^2 - 40x + 10x + 200 - 96 = 0$$

$$\begin{aligned}
 x^2 - 30x + 104 &= 0 \\
 (x - 26)(x - 4) &= 0 \\
 (x - 26) = 0 \quad (x - 4) &= 0 \\
 x = 26 \quad x &= 4
 \end{aligned}$$

The value of x is **4m**. It is not possible to be 26m because the total height of the figure is just 10m.

6. **C**

$$\text{Area}_{\text{square}} + \text{Area}_{\text{triangle}} = 24 \text{ square units}$$

$$s^2 + \frac{bh}{2} = 24 \text{ square units}$$

$$(8 - x)^2 + \frac{(8 - x)(x)}{2} = 24$$

$$2 \left[64 - 16x + x^2 + \left(\frac{8x - x^2}{2} \right) \right] = 24 \cdot 2$$

$$128 - 32x + 2x^2 + 8x - x^2 = 48$$

$$2x^2 - x^2 - 32x + 8x + 128 - 48 = 0$$

$$x^2 - 24x + 80 = 0$$

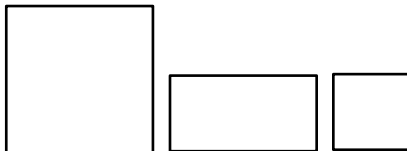
$$(x - 20)(x - 4) = 0$$

$$(x - 20) = 0 \quad (x - 4) = 0$$

$$x = 20 \quad x = 4$$

The value of x is **4m**. It is not possible to be 20m because the total height of the figure is just 8 m.

7. **D**



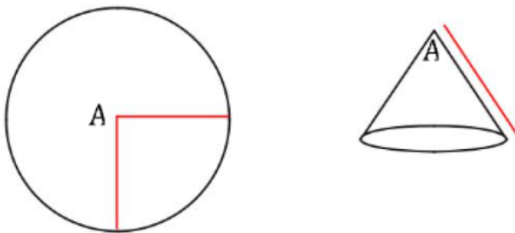
$$\text{Area}_{\text{rectangle}} = (\text{base})(\text{height})$$

$$\text{Area}_{\text{square paper}} = (16\text{cm})(16\text{cm})$$

$$\text{Area}_{\text{after first fold}} = (16\text{cm})(8\text{cm})$$

$$\text{Area}_{\text{after second fold}} = (8\text{cm})(8\text{cm}) = 64\text{cm}^2$$

8. **D**



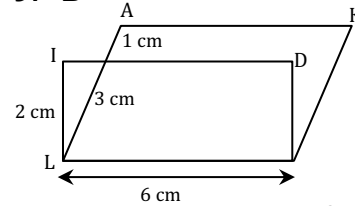
The area of the remaining portion of the circle is:

The surface area of the cone without a base is $\pi r l$ where l is the slant height of the cone, in this case, the old $r = 1$.

$$\pi r l = \pi \times r \times 1 =$$

$$r = \frac{3}{4}$$

9. **B**



$$A_{\text{RectangleLODI}} = 12\text{cm}^2$$

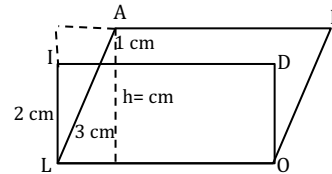
$$l \times w = 12\text{cm}^2$$

$$l \times 2 = 12$$

$$\frac{l}{2} = \frac{12}{2}$$

$$l = 6$$

Create a right triangle



$$x =$$

$$h = 2 \quad 3$$

$$h = 2 \quad 3$$

Using the concept of similar triangle by apply Ratio and Proportion (RAP)

$$\frac{2}{3} = \frac{x + 2}{4}$$

$$8 = 3x + 6$$

$$8 - 6 = 3x$$

$$\frac{2}{3} = \frac{3x}{3}$$

$$x = \frac{2}{3}$$

To get the height,

$$h = \frac{2}{3} + 2$$

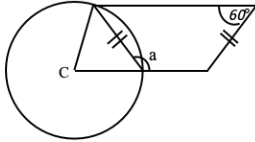
$$h = \frac{2+6}{3} = \frac{8}{3}$$

$$A_{\text{ParallelogramLOKA}} = b \times h$$

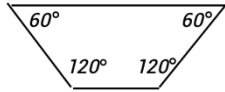
$$A_{\text{ParallelogramLOKA}} = 6 \times \frac{8}{3}$$

$$A_{\text{ParallelogramLOKA}} = \mathbf{16\text{cm}^2}$$

10. B



Since we have a trapezoid where two sides are equal,



The sum of the angles of a trapezoid is 360°

$$360 - 60 - 60 = 360 - 120 = 240$$

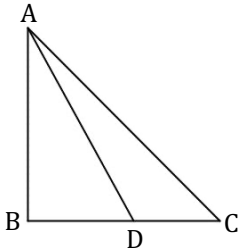
$$m\angle a = \frac{1}{2}(240)$$

$$m\angle a = 120$$

$$\frac{1}{2} m\angle a = \frac{120}{2} = 60^\circ$$

The measure of half of $\angle a$ is 60° .

11. A



Since the triangle is isosceles, $m\angle A = 45^\circ$.

$m\angle BAD = 45 - 15 = 30^\circ$. To get AB and BC, $BD = AD \sin 30$

$$4\sqrt{3} = AD \left(\frac{1}{2}\right)$$

$$AD = 8\sqrt{3}$$

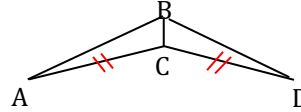
To get AB and BC,

$$AB = BC = AD \cos 30$$

$$= 8\sqrt{3} \left(\frac{\sqrt{3}}{2}\right) = (4 \times 3) = 12$$

The area is equal to $(12 \times 12) / 2 = \mathbf{72}$
square units.

12. C



An angle bisector divides the angle into two equal lengths.

Since \underline{BC} is a bisector of $\angle ABD$ and $\underline{AC} = \underline{CD}$,
 $\triangle ABC \cong \triangle DBC$